



Group – Theoretic Analysis in Balanced Incomplete Block Design Structures

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Abstract

The construction of block designs remains a challenging and open problem in combinatorial design theory, with balanced incomplete block designs (BIBDs) representing one of the most extensively studied classes. This research objective is to investigate the existence and structure of BIBDs from an algebraic perspective by examining their compliance with the axioms of finite groups. Both symmetric and non-symmetric BIBDs are considered, and relevant theorems are formulated and proved within the framework of finite group theory. The algebraic structures examined in this study fail to satisfy group axioms under the specified multiplicative operation. However, under addition, they satisfy the axioms of an abelian group, thereby establishing their algebraic compatibility with finite group structures.

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1.0 Introduction

A block design is a combinatorial arrangement represented by a pair (Y, A) , where Y denotes a finite set of elements and A is a collection of nonempty subsets of Y . The members of Y are known as points, whereas the subsets contained in A are referred to as blocks. The development of block designs is fundamentally concerned with the systematic allocation of treatments into blocks, which constitutes one of the central problems in combinatorial design theory. Over the past few decades, design theory has grown into a highly interdisciplinary research area,

attracting considerable attention from scholars in mathematics, computer science, engineering, applied mathematics, data science, and industrial research communities (Dukes, Esther & Richard, 2008; Colbourn, 2022; Stinson, 2022; Ding & Tang, 2023; Zhang & Lin, 2024). This expansion has largely been motivated by the increasing demand for efficient and well-structured designs capable of addressing challenges associated with large-scale computation, high-dimensional datasets, and complex systems.

Combinatorial design theory is a branch of combinatorial mathematics devoted to investigating the existence, construction, and characteristics of finite arrangements that satisfy specified balance and symmetry requirements. Among the most important and extensively researched structures within this discipline are Balanced Incomplete Block Designs (BIBDs) (Parathy, 2015; Street & Street, 2021). BIBDs and related configurations have found wide-ranging applications in error-correcting codes, cryptographic systems, information security, wireless and mobile communication networks, DNA screening through group-testing procedures, software and hardware reliability testing, interconnection network design, engineering optimization, and various biological and medical sciences (Mariusz, 2017; Tonchev, 2021; Colbourn & Rosa, 2023; Ding et al., 2022; Wu & Hamada, 2024). Within the framework of experimental design, BIBDs remain particularly valuable when practical constraints prevent complete randomization, especially in agricultural, industrial, and biomedical investigations (Mukerjee & Das, 2017; Nguyen & Tran, 2024).

Recent progress in computational linear algebra, symbolic computation, and simulation-based methodologies has substantially enhanced the practical relevance of BIBDs. These technological developments have enabled researchers to assess the robustness of such designs in the presence of missing observations, unequal error variances, and violations of classical model assumptions, thereby extending their applicability to contemporary data-analysis environments (Kageyama & Kubota, 2016; Mukerjee & Das, 2017; Preece & Bailey, 2021; Li & Wang, 2023). Furthermore, advances in high-performance computing have facilitated large-scale investigations involving design enumeration, isomorphism detection, Steiner quintuple design, and automorphism group characterization, thereby deepening the understanding of block design structures

(Colbourn, 2022; Akra et al., 2025; Isaac, 2026; Lin & Zhao, 2026).

BIBDs continue to present both theoretical and practical challenges. Existence conditions have not yet been completely resolved for many parameter combinations, and algebraic construction methods based on finite-field structures often require rigorous verification of balance and intersection properties across different parameter settings (Araujo & Pardo, 2022; Ding & Ling, 2024). In addition, contemporary applications increasingly require designs that are resilient to missing experimental units and capable of accommodating complex nuisance factors. Consequently, research efforts directed toward cyclic groups, novel procedures for generating N -point D-optimal symmetric and asymmetric designs, and the integration of combinatorial existence results with statistical robustness and computational efficiency remain active and important areas of investigation (Onyeka & Akra., 2024; Chen & Wei, 2025; Isaac et al., 2025, 2026).

The challenge of connecting combinatorial existence theory with the practical requirements of modern statistical applications therefore remains an important research frontier. Several scholars have contributed significantly to strengthening the algebraic and geometric foundations of BIBDs. Janardan (2018) employed Galois fields in the construction of BIBDs, while Neil (2010) examined classical construction approaches and Umoren & Isaac (2010) constructed N – Point D – Optimal designs from balanced confounded design. Awad and Banerjee (2013) provided a comprehensive review of BIBDs containing repeated blocks. Yu-pei et al. (2018) introduced innovative group divisible designs (GDDs), which constitute important generalizations of BIBDs. More recently, Akra et al. (2021, 2023, 2024, 2025, 2026), Muffat et al. (2025) and Isaac et al. (2025) have made notable contributions through algebraic, geometric, and optimality-oriented construction techniques, including t-

Steiner Quintuple Systems and methods based on finite Euclidean and projective geometries. Their investigations of isomorphisms and automorphisms have further revealed significant structural symmetries within these designs. Additional progress has also been reported in studies of difference sets, resolvable designs, and partially balanced designs (Singh & Dey, 2022; Kumar et al., 2023; Tang & Ding, 2024; Li et al., 2025).

Continued exploration in this field is important because it offers opportunities to integrate combinatorial design theory with algebraic frameworks such as finite groups, rings, and fields. Such integration has the potential to produce broader construction methodologies and provide deeper insight into the structural characteristics of designs. Establishing formal relationships between BIBDs and finite-group axioms not only enriches theoretical understanding but also supports practical algorithmic implementations in areas including coding theory, cryptography, and experimental design (Akra et al., 2025; Colbourn, 2022; Ding & Tang, 2023). According to Peter et al. (2008) combinatorial design theory and algebraic systems such as groups, finite fields, and geometrical structures are related.

Most studies construct BIBDs using groups but do not analyze how algebraic properties determine the overall structure of the design. Therefore, the present study seeks to investigate the existence of Balanced Incomplete Block Designs through algebraic approaches, with particular emphasis on determining whether these designs satisfy the axioms of finite group theory. The study therefore contributes toward addressing existing challenges and strengthening both the theoretical and practical relevance of algebraic methods in the construction of block designs.

2.0 Methodology

A Group – theoretic technique is employed by considering BIBDs as algebraic structures and

examining their conformity with the axioms of finite groups.

2.1 Finite Group

A finite group is a set equipped with a binary operation satisfying the following axioms:

1. For all $y_1, y_2 \in G$, then $y_1 * y_2 \in G$ is closed
2. For all $y_1, y_2, y_3 \in G$, $y_1 * (y_2 * y_3) = (y_1 * y_2) * y_3$ is associative
3. There exist an element $e \in G$, $\exists y_1 * e = y_1 = e * y_1, \forall y_1 \in G$, is identity.
4. For each $y_1 \in G$, \exists an element $y_1^{-1} \in G$ called an inverse of y_1 such that $y_1 * y_1^{-1} = e = y_1^{-1} * y_1$

A group is called a commutative or an abelian group if the following axiom is satisfied

5. For all $y_1, y_2 \in G$, $y_1 * y_2 = y_2 * y_1$ is commutative

A design G for a group $\langle G, * \rangle$ which the postulate (5) does not hold is called a non – abelian group. If G is finite, then $\langle G, * \rangle$ is called a finite group, otherwise, it is called an infinite group. A group $\langle G, * \rangle$ consisting of a nonempty set G and a binary operation $*$ on G is called a semigroup if the following axiom is satisfied:

$$S1: \forall y_1, y_2, y_3 \in G; y_1 * (y_2 * y_3) = (y_1 * y_2) * y_3$$

A semigroup is not always a group while the converse is true always. A design G on which a binary operation ($*$) is defined is called a groupoid. A semigroup with an identity element is called a monoid. A monoid in which every element has an inverse is called a group.

2.2 Subgroup

A design Y with a subset A of a group G is called a subgroup of G if A in a design Y is also a group under the same binary composition as

that in G . when A is a subgroup of G , we write $A \leq G$.

A group G is trivial subgroup of G if $A = G$ and $A = \{e\}$ such that $e \in G$ are both subgroup of G . If $A \leq G$ and $A \neq G$, $A \neq \{e\}$, then A is called a proper subgroup of G . For every $y_1, y_2 \in A$, $y_1 - y_2 \in A$, A is a subgroup of $(G, +)$, and for every $y_1, y_2 \in A$, $y_1 y_2^{-1} \in A$, A is a subgroup of $(G, *)$.

Let $y_1 \in G$ and let $\langle y_1 \rangle \subset G$ consisting of all powers of y_1 and their inverses. That is; $\langle y_1 \rangle = \{y_1^n / n \in Z\}$; if the binary operation in multiplication; or $\langle y_1 \rangle = \{y_1 n / n \in Z\}$; if the binary operation is addition. Hence $\langle y_1 \rangle$ is said to be the subgroup of G generated by the element y_1 and is also called a cyclic group of G . A group G is cyclic if $G = \langle y_1 \rangle$ for some elements $y_1 \in G$. For example, if Z is the set of all integers, then $(Z, +)$ is a cyclic group generated by the element 1, i.e. $(Z, +) = \langle 1 \rangle$.

3.0 Results/Implementation

Theorem 1

Let ψ be the family of designs and \cap be the operation of intersection of designs, then (ψ, \cap) is a commutative monoid.

Proof

Let ϕ and φ be a design of positive integers, then

(i) $\phi \cap \varphi \Rightarrow \phi, \varphi \in \psi$ is closed under \cap .

(ii) $\forall \phi, \varphi, \gamma \in \psi : (\phi \cap \varphi) \cap \gamma = \phi \cap (\varphi \cap \gamma)$. Therefore, associative law holds for operation \cap .

(iii) $(\phi \cap \varphi) = (\varphi \cap \phi)$ implies that $\phi, \varphi \in \psi$, then the commutative condition is satisfied.

(iv) For a null design $\theta \in \psi$ and $\phi \cap \theta = \phi$, therefore θ is the identity element. Hence, (ψ, \cap) is a commutative monoid.

Theorem 2

If a design G has a block b as an identity element, then the design defined as a group $(G, *)$ is unique.

Proof

Let a be identity element of a in G , then

$$\forall y_1 \in G: y_1 * a = a * y_1 = y_1 \quad (1)$$

$$\forall y_1 \in G: a' * y_1 = a' * y_1 = y_1 \quad (2)$$

Put $y_1 = a'$ in (1), we have

$$a' * a = a * a' = a' \quad (3)$$

Put $y_1 = a$ in (2), we have

$$a * a' = a' * a = a \quad (4)$$

Equate (3) and (4) imply

$$a = a'$$

Hence, the identity element of a design defined as a group $(G, *)$ is unique

Theorem 3

Let a and b be any two varieties in a BIBD defined as $a * b = a + b + ab$, then $(Z, *)$ is not a group, where Z is a set of integers.

Proof

For any $a, b \in Z \Rightarrow a * b = a + b + ab \in Z$.

Therefore Z is closed under $*$

$$\text{For } a, b, c \in Z, \quad \text{then} \quad (a * b) * c = (a + b + ab) * c$$

$$\Rightarrow (a + b + ab) + c + (a + b + ab)c \Rightarrow a + b + ab + a + ac + bc + abc \quad (5)$$

$$\text{Also } a * (b * c) = a * (b + c + bc) \Rightarrow a + (b + c + bc) + a(b + c + bc).$$

$$\Rightarrow a + b + c + bc + ab + ac + abc \quad (6)$$

Since (5) = (6), therefore Z is associative under $*$

Let $0 \in Z$. For any $a \in Z$, then $a * 0 = a + 0 + a \bullet 0 = a$ and $0 * a = 0 + a + 0 \bullet a = a$. Hence 0 is the identity element.

For

$$a * b = a + b + ab = 0 \Rightarrow a = \frac{b}{b-1}, b \geq 0, b \neq 1,$$

then $\frac{b}{b-1} \notin Z$ except $b = 0$ or 2 . Since not all integers have inverses. Hence $(Z, *)$ is not a group.

Example 1

Let (Y, A) be a BIBD where $(Y = Z_{15})$ and ω is a subset of Y , then (Y, A) is a semigroup and abelian group.

(a) Let $\omega_1, \omega_2, \dots, \omega_{15} \in Y$ and let $y_1, y_2, \dots, y_7 \in \omega_i$, then for any three elements in Y , say $y_1 = 3, y_2 = 5$ and $y_3 = 8$. It follows that:

(i) $x_1 * x_2 \in X, \forall x_1, x_2 \in Z_{15}$

$$0 \text{ mod } 15 \Rightarrow 0 \in Y$$

(ii) $(y_1 * y_2) * y_3 = y_1 * (y_2 * y_3)$

$$\forall y_1, y_2, y_3 \in Z_{15}, 0 \text{ mod } 15 \Rightarrow 0 \in Y$$

(iii) $y_2 * y_3 = y_3 * y_2, \forall y_2, y_3 \in Z_{15}$

$$10 \text{ mod } 15 \Rightarrow 10 \in Y$$

Table 1 demonstrate that the identity element and inverse axioms of group theory are not satisfied under the multiplicative operation.

Table 1: Multiplication Group table for integer modulo 15 ($Z_{15}, *$) design

| | | | | | | | | | | | | | | | |
|----|---|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
| * | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
| 2 | 0 | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 1 | 3 | 5 | 7 | 9 | 11 | 13 |
| 3 | 0 | 3 | 6 | 9 | 12 | 0 | 3 | 6 | 9 | 12 | 0 | 3 | 6 | 9 | 12 |
| 4 | 0 | 4 | 8 | 12 | 1 | 5 | 9 | 13 | 2 | 6 | 10 | 14 | 3 | 7 | 11 |
| 5 | 0 | 5 | 10 | 0 | 5 | 10 | 0 | 5 | 10 | 0 | 5 | 10 | 0 | 5 | 10 |
| 6 | 0 | 6 | 12 | 3 | 9 | 0 | 6 | 12 | 3 | 9 | 0 | 6 | 12 | 3 | 9 |
| 7 | 0 | 7 | 14 | 6 | 13 | 5 | 7 | 4 | 11 | 3 | 10 | 2 | 9 | 1 | 8 |
| 8 | 0 | 8 | 1 | 9 | 2 | 10 | 13 | 11 | 4 | 12 | 5 | 13 | 6 | 14 | 7 |
| 9 | 0 | 9 | 3 | 12 | 6 | 0 | 9 | 3 | 12 | 6 | 0 | 9 | 3 | 12 | 6 |
| 10 | 0 | 10 | 5 | 0 | 10 | 5 | 0 | 10 | 5 | 0 | 10 | 5 | 0 | 10 | 5 |
| 11 | 0 | 11 | 7 | 3 | 14 | 10 | 6 | 2 | 13 | 9 | 5 | 1 | 12 | 8 | 4 |
| 12 | 0 | 12 | 9 | 6 | 3 | 0 | 12 | 9 | 6 | 3 | 0 | 12 | 9 | 6 | 3 |
| 13 | 0 | 13 | 11 | 9 | 7 | 5 | 3 | 1 | 14 | 12 | 10 | 8 | 6 | 4 | 2 |
| 14 | 0 | 14 | 13 | 12 | 11 | 10 | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 1 |

Under the multiplication binary operation, BIBDs failed to satisfy the group axioms. Specifically: no identity element existed in some rows. Certain integers repeated within columns, preventing the existence of unique inverse. Consequently, the design structure

could not be classified as a group. But Y form a semigroup and abelian group.

Example 2

Suppose $(Y, +)$ and $(\omega's, +)$ be a BIB - design defined on the varieties $(Y = Z_{15})$ and

the blocks ($\omega's \in Y$), then the design $(Y, \omega's, +)$ is a group and an abelian group.

(a) Let $\omega_1, \omega_2, \dots, \omega_{15} \in Y$ and let $y_1, y_2, \dots, y_7 \in \omega's$ s.t $\omega's \subset Y$, selecting any three elements in X , say $y_1 = 3, y_2 = 5$ and $y_3 = 7$. It follows that:

(i) $y_1 + y_2 \in Z, y_1, y_2 \in Z_{15}$
 $\Rightarrow 8 \in Z_{15}$, therefore Z_{15} is closed under (+)

(ii) $y_1 + (y_2 + y_3) = (y_1 + y_2) + y_3$
 $\forall y_1, y_2, y_3 \in Z,$

$\Rightarrow 0 \in Z_{15}$, therefore Z_{15} is associative under (+)

(iii) for any $y_1, y_2 \in Z_{15}$

$y_1 + y_2 = y_2 + y_1 \Rightarrow 8 \in Z_{15}$, therefore Z_{15} is commutative under (+). Table 2 presents the verification of the identity element and inverse axioms of group theory are satisfied under the additive operation.

Table 2: Additive Group for integer modulo 15 ($Y, +$) design

| \oplus | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
|----------|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
| 0 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
| 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 0 |
| 2 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 0 | 1 |
| 3 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 0 | 1 | 2 |
| 4 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 0 | 1 | 2 | 3 |
| 5 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 0 | 1 | 2 | 3 | 4 |
| 6 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 0 | 1 | 2 | 3 | 4 | 5 |
| 7 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
| 8 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| 9 | 9 | 10 | 11 | 12 | 13 | 14 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| 10 | 10 | 11 | 12 | 13 | 14 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 11 | 11 | 12 | 13 | 14 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 12 | 12 | 13 | 14 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| 13 | 13 | 14 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| 14 | 14 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 |

Under the additive binary operation, BIBDs satisfied all group axioms. The operation was closed. Zero (0) served as the identity element and inverses were well – defined, for example - $0 = 0, -1 = 14, -2 = 13, -3 = 12, -4 = 11, -5 = 10, -6 = 9, -7 = 8, -8 = 7, -9 = 6, -10 = 5, -11 = 4, -12 = 3, -13 = 2, -14 = 1$. Associativity was preserved. Thus, the design structure formed a finite group under addition, specifically an abelian additive group.

From the above examples, the balanced incomplete block design is used: $X = \{1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F\}$

$$B = \left\{ \begin{array}{l} (123457), (12389AB), (14589CD), (2468ACE), \\ (123CDEF), (145ABEF), (2469BDF), (18967EF), \\ (28A57DF), (48C37BF), (3568BDE), (39A47DE), \\ (59C27BE), (6AC17BD), (3659ACF) \end{array} \right\}$$

4.0 Discussion

The results of this study demonstrate that although Balanced Incomplete Block Designs are fundamentally combinatorial structures, they possess significant algebraic characteristics when examined from a group-theoretic perspective. The analysis reveals that BIBDs do not satisfy the requirements of finite groups under multiplicative operations. However, they fulfill the necessary axioms to form abelian groups under addition. These findings are consistent

with earlier investigations that highlighted the strong relationship between combinatorial design theory and algebraic systems such as groups, finite fields, and geometrical structures (Peter et al., 2008). By establishing the additive group properties of BIBDs, this research provides further insight into their algebraic foundations within the framework of finite group theory. The findings strengthen the theoretical connection between combinatorial designs and abstract algebra while offering a basis for future studies aimed at developing new BIBD constructions through group-theoretic methods.

5.0 Conclusion

This study has established that Balanced Incomplete Block Designs do not constitute finite groups when considered under multiplicative binary operations. Nevertheless, the investigation confirms that BIBDs satisfy the axioms of finite groups under additive binary operations and consequently form abelian additive groups irrespective of the specific design configuration. The proofs developed in this work are consistent with fundamental algebraic concepts, including commutative monoids, uniqueness properties, and integer-based structural characteristics inherent in the designs.

The study provides preliminary evidence of algebraic relationships between the investigated structures and finite group concepts. By strengthening the connection between combinatorial design theory and abstract algebra, the findings contribute to a deeper understanding of the mathematical foundations of BIBDs.

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